THE ACCURACY OF TAI

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Abstract

By definition, the duration of the TAI scale unit should be as close as possible to the SI second on the rotating geoid. The frequency of the free atomic time scale, computed at the BIPM as a weighted average of commercial clock data obtained from time laboratories, was carefully steered for more than ten years in order to generate an international time reference which conforms with this definition. In 1995, uniform application of the correction compensating for the black-body frequency shift in primary frequency standards artificially degraded the accuracy of TAI. A procedure to compensate for this effect was immediately implemented, but only compensated for the natural drift of the free atomic time scale over the first two years, leading to the uncomfortable condition that TAI was not sufficiently accurate. Since April 1997, an improvement in the accuracy of TAI has been detected, but has still to be confirmed. For this purpose results from very accurate primary frequency standards, such as the new cesium fountains now under development, would be very helpful.

INTRODUCTION

From a declaration of the Comité Consultatif pour la Définition de la Seconde, CCDS, in 1980 [1] International Atomic Time, TAI, is

a coordinate time scale defined in a geocentric reference frame with the SI second as realized on the rotating geoid as the scale unit.

Since 1 January 1988, the Bureau International des Poids et Mesures, BIPM, has been responsible for the generation and dissemination of TAI, which is a world-wide reference time scale and thus should be maintained in such a way as to be as reliable, stable and accurate as possible.

TAI relies on measurements taken on a regular basis from commercial atomic clocks and primary frequency standards maintained in national timing centers spread world-wide. Reliability and stability are optimized in the first step of the procedure used for combining these data. In this, a free atomic time scale, EAL (échelle atomique libre), is computed as a weighted average of data from a large number of free-running and independent atomic clocks [2]. The weight attributed to a given contributing clock increases with its stability which makes it possible to improve the stability of the resulting average (over the period January 1996 - August 1997, the stability of EAL was characterized by an Allan deviation close to 1×10^{-15} for averaging times of 40 d [3]). No attempt is made to ensure the conformity of the EAL scale unit with the SI second as realized on the rotating geoid.

In a second step, the duration of the scale unit of EAL is evaluated by comparison with data from primary cesium standards which realize the SI second. TAI is then derived from EAL by adding a linear function of time with a slope designed to bring the TAI scale unit close to the SI second on the surface of the rotating geoid. To maintain accuracy the frequency offset between TAI and EAL is changed when necessary [4], the magnitude of the changes being of the same order as the frequency fluctuations resulting from the instability of EAL. This operation is referred to as the 'steering of TAI'.

In the first section of this paper we compare the frequency of EAL, over the last twelve years, to the frequencies of the two continuously operating primary frequency standards PTB CS1 and PTB CS2. We also justify the frequency steering corrections which were applied over this period. The second section describes how the accuracy of TAI is estimated and, in the third section, comments are given on individual and global estimations of the accuracy of TAI for the period January 1996 - October 1997.

FREQUENCY OF EAL SINCE 1985

Figure 1 shows the frequency of the free atomic time scale EAL, computed over successive two-month intervals, relative to the continuously operating primary frequency standards PTB CS1 and PTB CS2, maintained at the PTB, Braunschweig, Germany. It includes data taken since January 1985 when only PTB CS1 was in operation. First measurements from PTB CS2 were provided in 1986. PTB CS1 ceased operation in March 1995. The type B standard uncertainties of PTB CS1 and PTB CS2 are respectively 3×10^{-14} and 1.5×10^{-14}. The dashed line added to Fig. 1 shows the frequency of EAL relative to TAI, each step corresponding to an additive steering correction.

The step observed in the values relative to PTB CS2 in April 1995 comes from the application of a correction compensating for the black-body frequency shift experienced by the standard: under the influence of the radiation from the walls surrounding the atoms inside the clock at temperature T, the clock transition frequency is reduced with respect to its value at $T = 0$ K. From the formula provided by Itano *et al.* [5] the amplitude of the effect is $1{,}7 \times 10^{-14}$ for $T = 299$ K, with an uncertainty conservatively estimated at 1×10^{-15}. The uniform application of this correction to results provided by primary frequency standards is recommended by the Comité Consultatif pour la Définition de la Seconde [6].

Figure 1 calls for some remarks:

- The values relative to PTB CS1 and PTB CS2 agree within their uncertainty bars, but a systematic difference of about 2×10^{-14} may be observed between them.
- The instability of the frequency values of EAL relative to PTB CS1 may be entirely due to the intrinsic instability of the standard. Alternatively, one may note that the dispersion of the values relative to PTB CS2 falls so that it approaches the intrinsic instability of the standard at the beginning of 1995.
- If we except the step due to the application of the black-body correction from April 1995, one can distinguish two successive phases in the behavior of EAL relatively to the primary frequency standards:

    a) for the period January 1985 - June 1993, EAL presented a drift of about 1.5×10^{-14}/y,

    b) from mid-1993, the amplitude of the drift has been much smaller.

  The epoch separating these phases corresponds to that at which replacement of clocks of older design by the new HP 5071A units began to take place in the time laboratories contributing to TAI.
- The first drift was compensated by a series of 12 cumulative frequency steering corrections, each of amplitude 5×10^{-15}, applied between 1989 and 1992.

- The application of the black-body correction has required the implementation of a compensation procedure starting at the beginning of 1995, and which still continues. This takes the form of cumulative steering corrections applied at 60-day intervals and of increasing amplitude (1×10^{-15} from January 1995 to June 1996, 1.5×10^{-15} from July 1996 to February 1997, and 2×10^{-15} starting March 1997).

The following conclusions can be drawn for the long-term behavior of the EAL frequency:

- In the period 1985-1992, the time scale EAL presented a nearly constant frequency drift which was compensated through cumulative steering corrections between 1989 and 1992.
- Since 1993 the introduction of HP 5071A clocks has reduced the drift of EAL and no other frequency corrections have been applied, except those which compensate the additional step due to the application of the black-body correction.
- The procedure for compensating the black-body step, accelerated in 1996, now progresses faster than the natural drift of EAL.

It follows that over the last 12 years the applied frequency steering corrections have been appropriate and have contributed to improvement of the accuracy of TAI. The studies presented in the following focus on more recent years; they cover the period January 1996 - October 1997.

HOW CAN WE ESTIMATE THE ACCURACY OF TAI?

From its definition, the duration of the TAI scale unit, u, should be as close as possible to the duration of the SI second, u_0, on the rotating geoid. The accuracy of TAI may thus be characterized by the relative departure, d, defined as:

$$d = \frac{u - u_0}{u_0}, \qquad (1)$$

and its uncertainty σ.

In (1) u is expressed in seconds and $u_0 = 1$ s is the period of the 1 Hz signal provided by a primary frequency standard, after all frequency corrections, especially those compensating for the black-body shift and the gravitational red shift, have been applied.

Individual values d_i of d are provided by comparing the frequency of TAI with that of primary frequency standard i over a given time interval I_{Ci}, designed as the calibration interval and defined by its length τ_{Ci} and its central date t_{Ci}. The corresponding uncertainty, σ_i, is generally close to the type B uncertainty (1 σ) of the primary standard, σ_{Bi}, however, it may happen that the uncertainty introduced by the transfer to TAI (local comparison inside the laboratory and GPS transfer) is not negligible when compared to σ_{Bi} and should be accounted for.

The individual estimations can also be treated in a global way in order to deliver a more precise value of d for any interval of estimation I_E, defined by its length τ_E and its central date t_E. This global treatment operates with preceding and following calibrations taking place over intervals I_{Ci} included or not in I_E, even partially overlapping with I_E, and for which $t_E - t_{Ci}$ may be positive or negative. It is thus necessary to transfer the individual calibrations (d_i, σ_i) temporally over $|t_E - t_{Ci}|$. In the temporal transfer the values d_i are kept constant but the values σ_i are increased. In fact, the temporal transfer is carried out by the time scale itself, which, because it is continuous, can temporally link calibration and estimation intervals. Its instability over $|t_E - t_{Ci}|$ thus creates a new component of uncertainty.

After temporal transfer to the chosen interval I_E, calibrations are combined to obtain d and σ valid over I_E. In practice, this global treatment is not an easy task because the time scale is affected by different types of noise according to the length of time involved, and because the system is not stationary since the time scale stability improves with the passing of time. In addition, the combination of transferred calibrations should be optimized in order to obtain the best global estimate (with the smallest possible uncertainty). This problem was solved in 1977 by Azoubib, Granveaud and Guinot [7], the question of temporal transfer is also treated in [8]. Post-processing allows d to be estimated over a given I_E through a weighted average of all individual measurements which have occurred before and after t_E. An expression of the error made in the estimation process is given in [7] (page 89). It involves the weights and a number of other parameters:

- the uncertainty σ_i of each measurement d_i,
- a model for the stability of the time scale,
- the length of individual calibration intervals τ_{Ci},
- the length of the estimation interval τ_E,
- the length of the time interval $|t_E - t_{Ci}|$ separating the calibration intervals and the estimation interval.

Minimizing this error makes it possible to determine the weights (the sum of which is equal to 1) and to obtain the value of σ which corresponds to the computed minimum error.

The method described in [7] is applied at the BIPM and provides the regular estimates of d and σ which are published in successive issues of *Circular T* and of the *Annual Report of the BIPM Time Section*. We give results of this treatment for the period January 1996 - October 1997 in the following section.

ESTIMATION OF TAI ACCURACY SINCE JANUARY 1996

Since January 1996, individual measurements of the TAI frequency have been provided by five primary frequency standards:

- LPTF-FO1, which is a cesium fountain developed at the BNM-LPTF, Paris, France [9]. The preliminary evaluation of its accuracy led to $\sigma_{B\ FO1} = 3\times10^{-15}$, a value never reached before. Three measurements, taken in May 1996 and averaged over $\tau_{C\ FO1} \approx 10$ h, were sent to the BIPM.
- NIST-7, which is the optically pumped primary frequency standard developed at the NIST, Boulder, Colorado, USA [10]. In the period covered by this report, it provided five measurements which cover a 5-day period in March 1996 and four 10-day periods in May 1996, December 1996, June 1997 and October 1997; $\sigma_{B\ NIST\text{-}7} = 1\text{x}10^{-14}$ for the first two and the last measurements, $7\text{x}10^{-15}$ for the third and fourth measurements.
- PTB CS2 and PTB CS3, which are classical primary frequency standards operating continuously as clocks at the PTB, Braunschweig, Germany. Frequency measurements are taken continuously and are used over successive two-month periods; $\sigma_{B\ PTB\ CS2} = 1.5\text{x}10^{-14}$, except for the measurement covering the period July-August 1997 for which $\sigma_{B\ PTB\ CS2} = 2.7\text{x}10^{-14}$, and $\sigma_{B\ PTB\ CS3} = 1.4\text{x}10^{-14}$.
- SU MCsR 102, which is a classical primary frequency standard operated at the VNIIFTRI, Moscow, Russia. It delivered two measurements, both averaged over two-month periods, in February and March 1996; $\sigma_{B\ MCsR} = 5\text{x}10^{-14}$.

Values of d_i deduced from these individual measurements are given in Fig. 2. The corresponding uncertainties σ_i are close to σ_{Bi} except for points originating from LPTF-FO1 data, for which an additional uncertainty of about $5\text{x}10^{-15}$ must be taken into account for the link to TAI.

Estimates of d obtained by global treatment of individual measurements are added as a continuous line in Fig. 1. The chosen intervals of estimation I_E correspond to successive two-month periods in phase with the calendar bimesters. For the year 1997, the uncertainty σ on d values is about 1×10^{-14}.

Comments on Fig. 2 are as follows:

- The transfer of the LPTF-FO1 measurements to TAI is not optimized and it has not produced new results since May 1996: TAI thus does not take advantage of its outstanding accuracy.
- The relatively high uncertainty of the primary standard SU MCsR 102 makes its weight very low in the estimation of values for d.
- Measurements from PTB CS3 are not used in the computation of d values because this standard experienced frequency steps of several parts in 10^{14} over the period under study.
- Results provided by PTB CS2 show a high stability except for the measurement covering the two month interval July-August 1997 over which $d_{\mathrm{PTB\ CS2}}$ experienced a step of about 2×10^{-14}. The PTB informed the BIPM that this measurement should be used with a higher value of type B uncertainty. If we except this particular measurement, we observe a decrease of the duration of the TAI scale unit when compared to the SI second as produced on the rotating geoid by PTB CS2 in September-October 1997: $d_{\mathrm{PTB\ CS2}} = 1.4 \times 10^{-14}$ and $\sigma_{\mathrm{PTB\ CS2}} = 1.5 \times 10^{-14}$.
- Measurements from NIST-7 cover five or ten day intervals and have calibration dates spaced at nearly equal intervals of six months. The last three results seem to indicate a decreasing trend in the relative duration of the TAI scale unit. The decrease progresses much faster than that observed in the PTB CS2 results, so the two standards are not in such close agreement as they were before January 1997. Nevertheless, they agree within their uncertainty bars.

Over the period under study, estimates of TAI accuracy mainly relied on measurements provided by two primary frequency standards PTB CS2 and NIST-7. These two provide results on a regular basis and so are particularly helpful for the estimation of TAI accuracy. The situation is thus comparable with the one we experienced earlier when we had at our disposal both PTB CS1 and PTB CS2, but with a greatly improved level of stability in the data.

Until April 1997, the average d value was close to 2×10^{-14}, a discrepancy nearly equal to that resulting from uniform application of the correction for the black-body radiation frequency shift in 1995. The successive frequency steering corrections thus simply compensated for the natural drift of the scale. The accuracy of TAI was thus not as good as desired so we progressively reinforced the amplitude of the frequency steering correction from 1×10^{-15} to 1.5×10^{-15} and 2×10^{-15} (see the second section of the paper).

Since April 1997, it appears that the duration of the TAI scale unit is decreasing as if the steering frequency corrections were progressively compensating the 'black-body' step. At first sight the accuracy of TAI may be thought to be improving, but the trend is still slight and we must wait for new measurements from PTB CS2 and NIST-7 to confirm it. In addition, the uncertainty σ affecting the values of d is still rather large, 1×10^{-14}, and is difficult to reduce given the type B uncertainties of these two standards. Results from new cesium fountains under development in a number of laboratories should confirm the decreasing trend of d values and also reduce the uncertainty in these values.

CONCLUSIONS

The accuracy of TAI is estimated by computation of the relative departure d, together with its uncertainty σ, of the duration of its scale unit from the SI second as produced by primary frequency standards on the rotating geoid. The frequency of the free atomic time scale, computed at the BIPM as a weighted average of commercial clock data provided by time laboratories, was carefully steered for more than ten years in order to generate an international atomic time which conforms with its definition. At the end of 1994, $d = 0.2 \times 10^{-14}$ and $\sigma = 2.0 \times 10^{-14}$.

In 1995, the uniform application of the correction compensating for the black-body frequency shift in primary frequency standards artificially degraded the accuracy of TAI: over May - June 1995, $d = 2.3 \times 10^{-14}$ and $\sigma = 1.0 \times 10^{-14}$. A procedure to compensate for this effect was immediately implemented, but, over the first two years, merely compensated for the natural drift of the free atomic time scale, leading to the uncomfortable conclusion that TAI was not sufficiently accurate.

Since April 1997, an improvement in TAI accuracy has been observed: over September - October 1997, $d = 1.2 \times 10^{-14}$ and $\sigma = 1.0 \times 10^{-14}$, but this has still to be confirmed. For this purpose data from the very accurate primary frequency standards, among them the new cesium fountains now in development in time laboratories, would be very helpful.

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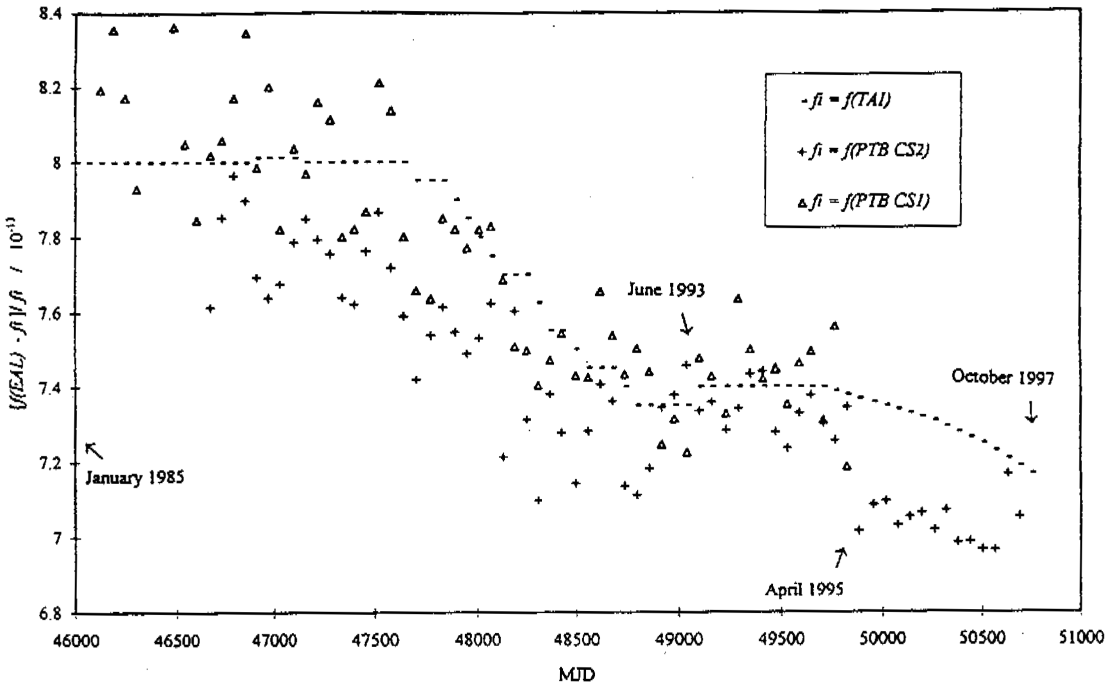

Figure 1. Frequency of the free atomic time scale EAL relative to PTB CS1, PTB CS2 and TAI over the last 12 years.

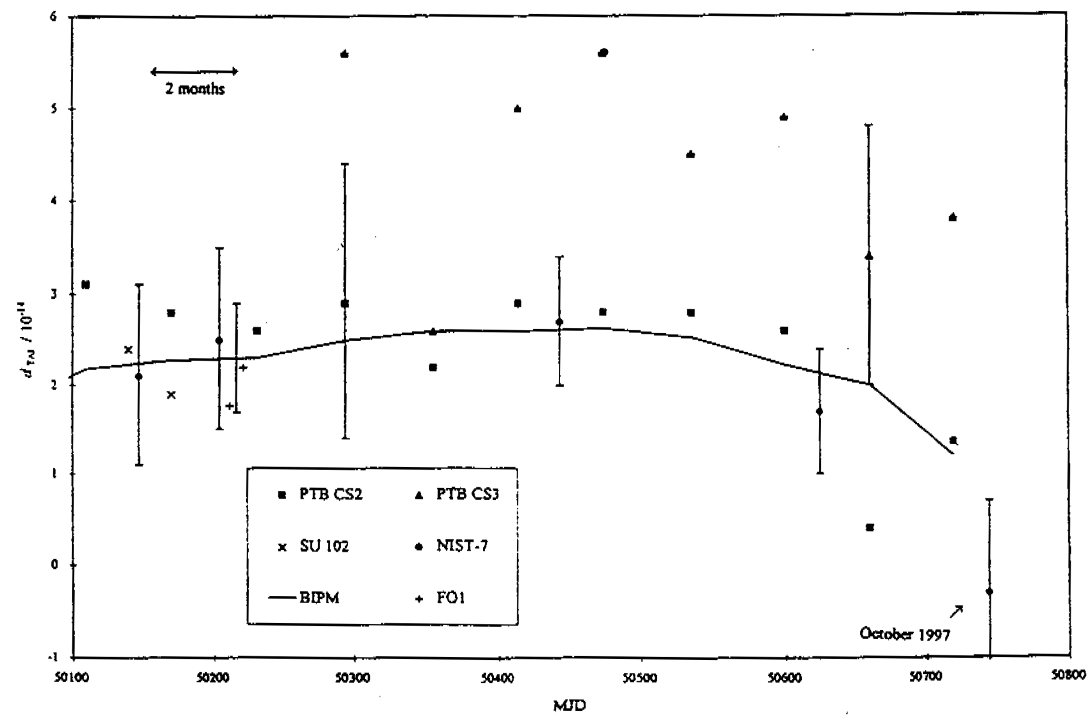

Figure 2. Estimates of the relative departure d of the TAI scale unit from the SI second as produced by primary frequency standards on the rotating geoid. The black curve corresponds to the BIPM global estimate.

Questions and Answers

BERNARD GUINOT (OBSERVATOIRE de PARIS): You mentioned the need of other primary standards. I would say that there should be a need of other operating primary standards. The pity is that there are some standards which are not really used in forming the current measurement of the frequency of TAI; it was already recommended that the standard as a primary standard be operated on the operational basis. Could we insist on the importance of that?

CLAUDINE THOMAS (BIPM): Yes, thank you. It is true that the BIPM needs measurements. But what we need is regular measurements. For instance, the example of NIST-7 is very important. We have measurements, I would say, every four or five months, but it comes regularly. So each time it comes, we have new information and this is very helpful for us. One measurement, even very accurate, is not very useful if it does not come on a regular basis. That is something which is very important for us. Thank you for this remark.